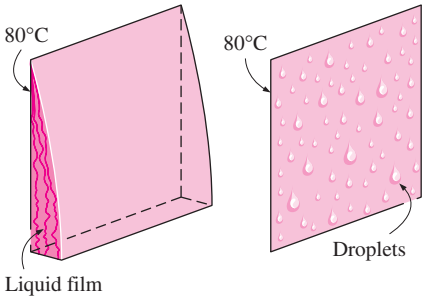
in the two-phase flow (bubbly flow, slug-flow, annular-flow, and mist-flow) cases, but they are beyond the scope of this introductory text. A crude estimate for heat flux in flow boiling can be obtained by simply adding the forced convection and pool boiling heat fluxes.

10–4 ▪ CONDENSATION HEAT TRANSFER

Condensation occurs when the temperature of a vapor is reduced *below* its saturation temperature T_{sat}. This is usually done by bringing the vapor into contact with a solid surface whose temperature T_s is *below* the saturation temperature T_{sat} of the vapor. But condensation can also occur on the free surface of a liquid or even in a gas when the temperature of the liquid or the gas to which the vapor is exposed is below T_{sat}. In the latter case, the liquid droplets suspended in the gas form a fog. In this chapter, we will consider condensation on solid surfaces only.

Two distinct forms of condensation are observed: *film condensation* and *dropwise condensation.* In **film condensation,** the condensate wets the surface and forms a liquid film on the surface that slides down under the influence of gravity. The thickness of the liquid film increases in the flow direction as more vapor condenses on the film. This is how condensation normally occurs in practice. In **dropwise condensation,** the condensed vapor forms droplets on the surface instead of a continuous film, and the surface is covered by countless droplets of varying diameters (Fig. 10–20).



(*a*) Film condensation (*b*) Dropwise condensation

FIGURE 10–20
When a vapor is exposed to a surface at a temperature below T_{sat}, condensation in the form of a liquid film or individual droplets occurs on the surface.

In film condensation, the surface is blanketed by a liquid film of increasing thickness, and this "liquid wall" between solid surface and the vapor serves as a *resistance* to heat transfer. The heat of vaporization h_{fg} released as the vapor condenses must pass through this resistance before it can reach the solid surface and be transferred to the medium on the other side. In dropwise condensation, however, the droplets slide down when they reach a certain size, clearing the surface and exposing it to vapor. There is no liquid film in this case to resist heat transfer. As a result, heat transfer rates that are more than 10 times larger than those associated with film condensation can be achieved with dropwise condensation. Therefore, dropwise condensation is the preferred mode of condensation in heat transfer applications, and people have long tried to achieve sustained dropwise condensation by using various vapor additives and surface coatings. These attempts have not been very successful, however, since the dropwise condensation achieved did not last long and converted to film condensation after some time. Therefore, it is common practice to be conservative and assume film condensation in the design of heat transfer equipment.

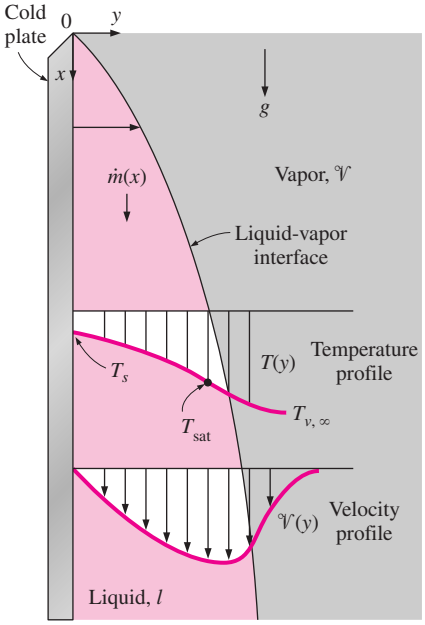


FIGURE 10–21
Film condensation on a vertical plate.

10–5 ▪ FILM CONDENSATION

We now consider film condensation on a vertical plate, as shown in Figure 10–21. The liquid film starts forming at the top of the plate and flows downward under the influence of gravity. The thickness of the film δ *increases* in the flow direction x because of continued condensation at the liquid–vapor interface. Heat in the amount h_{fg} (the latent heat of vaporization) is released during condensation and is *transferred* through the film to the plate surface at temperature T_s. Note that T_s must be below the saturation temperature T_{sat} of the vapor for condensation to occur.

Typical velocity and temperature profiles of the condensate are also given in Figure 10–21. Note that the *velocity* of the condensate at the wall is zero because of the "no-slip" condition and reaches a *maximum* at the liquid–vapor interface. The *temperature* of the condensate is T_{sat} at the interface and decreases gradually to T_s at the wall.

As was the case in forced convection involving a single phase, heat transfer in condensation also depends on whether the condensate flow is *laminar* or *turbulent*. Again the criterion for the flow regime is provided by the Reynolds number, which is defined as

$$\text{Re} = \frac{D_h \rho_l \mathcal{V}_l}{\mu_l} = \frac{4 A_c \rho_l \mathcal{V}_l}{p\mu_l} = \frac{4 \rho_l \mathcal{V}_l \delta}{\mu_l} = \frac{4\dot{m}}{p\mu_l} \qquad \textbf{(10-8)}$$

where

$D_h = 4A_c/p = 4\delta$ = hydraulic diameter of the condensate flow, m

p = wetted perimeter of the condensate, m

$A_c = p\delta$ = wetted perimeter × film thickness, m^2, cross-sectional area of the condensate flow at the lowest part of the flow

ρ_l = density of the liquid, kg/m^3

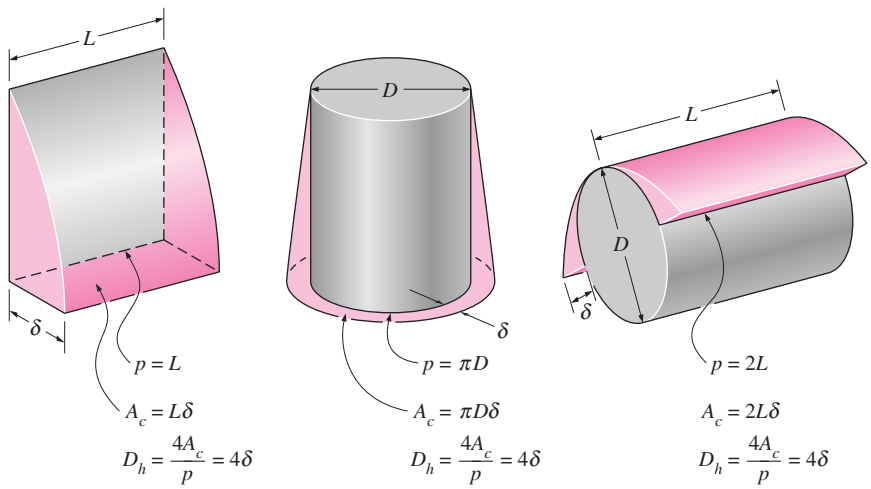
μ_l = viscosity of the liquid, kg/m · s

$\mathcal{V}$ = average velocity of the condensate at the lowest part of the flow, m/s

$\dot{m} = \rho_l \mathcal{V}_l A_c$ = mass flow rate of the condensate at the lowest part, kg/s

The evaluation of the hydraulic diameter D_h for some common geometries is illustrated in Figure 10–22. Note that the hydraulic diameter is again defined such that it reduces to the ordinary diameter for flow in a circular tube, as was done in Chapter 8 for internal flow, and it is equivalent to 4 times the thickness of the condensate film at the location where the hydraulic diameter is evaluated. That is, $D_h = 4\delta$.

The latent heat of vaporization h_{fg} is the heat released as a unit mass of vapor condenses, and it normally represents the heat transfer per unit mass of condensate formed during condensation. However, the condensate in an actual



(*a*) Vertical plate (*b*) Vertical cylinder (*c*) Horizontal cylinder

FIGURE 10–22
The wetted perimeter p, the condensate cross-sectional area A_c, and the hydraulic diameter D_h for some common geometries.

condensation process is cooled further to some average temperature between T_{sat} and T_s, releasing *more heat* in the process. Therefore, the actual heat transfer will be larger. Rohsenow showed in 1956 that the cooling of the liquid below the saturation temperature can be accounted for by replacing h_{fg} by the **modified latent heat of vaporization** h_{fg}^*, defined as

$$h_{fg}^* = h_{fg} + 0.68C_{pl}\,(T_{sat} - T_s) \tag{10-9a}$$

where C_{pl} is the specific heat of the liquid at the average film temperature.

We can have a similar argument for vapor that enters the condenser as **superheated vapor** at a temperature T_v instead of as saturated vapor. In this case the vapor must be cooled first to T_{sat} before it can condense, and this heat must be transferred to the wall as well. The amount of heat released as a unit mass of superheated vapor at a temperature T_v is cooled to T_{sat} is simply $C_{pv}(T_v - T_{sat})$, where C_{pv} is the specific heat of the vapor at the average temperature of $(T_v + T_{sat})/2$. The modified latent heat of vaporization in this case becomes

$$h_{fg}^* = h_{fg} + 0.68C_{pl}\,(T_{sat} - T_s) + C_{pv}\,(T_v - T_{sat}) \tag{10-9b}$$

With these considerations, the rate of heat transfer can be expressed as

$$\dot{Q}_{conden} = hA_s(T_{sat} - T_s) = mh_{fg}^* \tag{10-10}$$

where A_s is the heat transfer area (the surface area on which condensation occurs). Solving for $\dot{m}$ from the equation above and substituting it into Eq. 10–8 gives yet another relation for the Reynolds number,

$$\mathrm{Re} = \frac{4\dot{Q}_{conden}}{p\mu_l\, h_{fg}^*} = \frac{4A_s\, h(T_{sat} - T_s)}{p\mu_l\, h_{fg}^*} \tag{10-11}$$

This relation is convenient to use to determine the Reynolds number when the condensation heat transfer coefficient or the rate of heat transfer is known.

The temperature of the liquid film varies from T_{sat} on the liquid–vapor interface to T_s at the wall surface. Therefore, the properties of the liquid should be evaluated at the *film temperature* $T_f = (T_{sat} + T_s)/2$, which is approximately the *average* temperature of the liquid. The h_{fg}, however, should be evaluated at T_{sat} since it is not affected by the subcooling of the liquid.

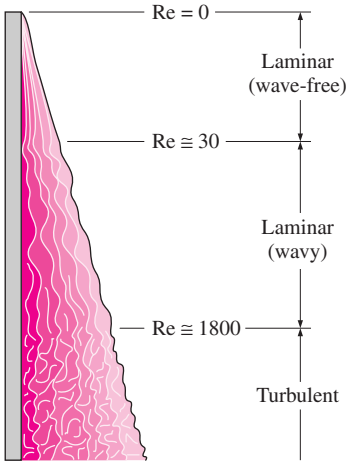


FIGURE 10–23
Flow regimes during film condensation on a vertical plate.

Flow Regimes

The Reynolds number for condensation on the outer surfaces of vertical tubes or plates increases in the flow direction due to the increase of the liquid film thickness δ. The flow of liquid film exhibits *different regimes,* depending on the value of the Reynolds number. It is observed that the outer surface of the liquid film remains *smooth* and *wave-free* for about Re ≤ 30, as shown in Figure 10–23, and thus the flow is clearly *laminar.* Ripples or waves appear on the free surface of the condensate flow as the Reynolds number increases, and the condensate flow becomes fully *turbulent* at about Re ≈ 1800. The condensate flow is called *wavy-laminar* in the range of 450 < Re < 1800 and *turbulent* for Re > 1800. However, some disagreement exists about the value of Re at which the flow becomes wavy-laminar or turbulent.

Heat Transfer Correlations for Film Condensation

Below we discuss relations for the average heat transfer coefficient h for the case of *laminar* film condensation for various geometries.

1 Vertical Plates

Consider a vertical plate of height L and width b maintained at a constant temperature T_s that is exposed to vapor at the saturation temperature T_{sat}. The downward direction is taken as the positive x-direction with the origin placed at the top of the plate where condensation initiates, as shown in Figure 10–24. The surface temperature is below the saturation temperature ($T_s < T_{sat}$) and thus the vapor condenses on the surface. The liquid film flows downward under the influence of gravity. The film thickness δ and thus the mass flow rate of the condensate increases with x as a result of continued condensation on the existing film. Then heat transfer from the vapor to the plate must occur through the film, which offers resistance to heat transfer. Obviously the thicker the film, the larger its thermal resistance and thus the lower the rate of heat transfer.

The analytical relation for the heat transfer coefficient in film condensation on a vertical plate described above was first developed by Nusselt in 1916 under the following simplifying assumptions:

1. Both the plate and the vapor are maintained at *constant temperatures* of T_s and T_{sat}, respectively, and the temperature across the liquid film varies *linearly*.
2. Heat transfer across the liquid film is by pure *conduction* (no convection currents in the liquid film).
3. The velocity of the vapor is low (or zero) so that it exerts *no drag* on the condensate (no viscous shear on the liquid–vapor interface).
4. The flow of the condensate is *laminar* and the properties of the liquid are constant.
5. The acceleration of the condensate layer is negligible.

Then Newton's second law of motion for the volume element shown in Figure 10–24 in the vertical x-direction can be written as

$$\sum F_x = ma_x = 0$$

since the acceleration of the fluid is zero. Noting that the only force acting downward is the weight of the liquid element, and the forces acting upward are the viscous shear (or fluid friction) force at the left and the buoyancy force, the force balance on the volume element becomes

$$F_{\text{downward}\downarrow} = F_{\text{upward}\uparrow}$$

$$\text{Weight} = \text{Viscous shear force} + \text{Buoyancy force}$$

$$\rho_l g(\delta - y)(bdx) = \mu_l \frac{du}{dy}(bdx) + \rho_v g(\delta - y)(bdx)$$

Canceling the plate width b and solving for du/dy gives

$$\frac{du}{dy} = \frac{g(\rho_l - \rho_v)g(\delta - y)}{\mu_l}$$

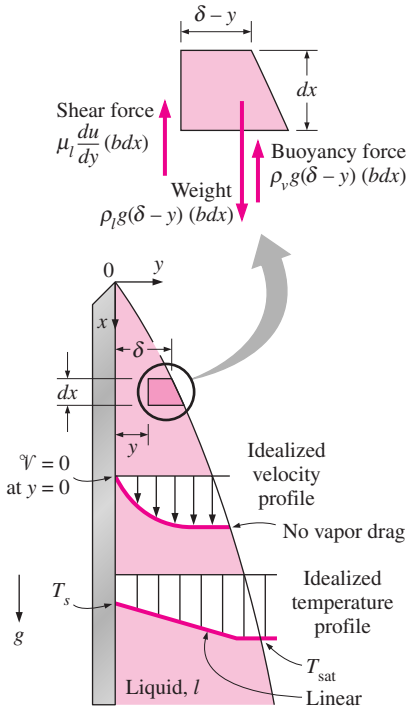


FIGURE 10–24
The volume element of condensate on a vertical plate considered in Nusselt's analysis.

Integrating from $y = 0$ where $u = 0$ (because of the no-slip boundary condition) to $y = y$ where $u = u(y)$ gives

$$u(y) = \frac{g(\rho_l - \rho_v)g}{\mu_l}\left(y\delta - \frac{y^2}{2}\right) \tag{10-12}$$

The mass flow rate of the condensate at a location x, where the boundary layer thickness is δ, is determined from

$$\dot{m}(x) = \int_A \rho_l u(y)dA = \int_{y=0}^{\delta} \rho_l u(y)bdy \tag{10-13}$$

Substituting the $u(y)$ relation from Equation 10–12 into Eq. 10–13 gives

$$\dot{m}(x) = \frac{gb\rho_l(\rho_l - \rho_v)\delta^3}{3\mu_l} \tag{10-14}$$

whose derivative with respect to x is

$$\frac{d\dot{m}}{dx} = \frac{gb\rho_l(\rho_l - \rho_v)\delta^2}{\mu_l}\frac{d\delta}{dx} \tag{10-15}$$

which represents the rate of condensation of vapor over a vertical distance dx. The rate of heat transfer from the vapor to the plate through the liquid film is simply equal to the heat released as the vapor is condensed and is expressed as

$$d\dot{Q} = h_{fg}\,d\dot{m} = k_l(bdx)\frac{T_{\text{sat}} - T_s}{\delta} \quad \rightarrow \quad \frac{d\dot{m}}{dx} = \frac{k_l\,b}{h_{fg}}\frac{T_{\text{sat}} - T_s}{\delta} \tag{10-16}$$

Equating Eqs. 10–15 and 10–16 for $d\dot{m}/dx$ to each other and separating the variables give

$$\delta^3\,d\delta = \frac{\mu_l\,k_l(T_{\text{sat}} - T_s)}{g\rho_l(\rho_l - \rho_v)h_{fg}}\,dx \tag{10-17}$$

Integrating from $x = 0$ where $\delta = 0$ (the top of the plate) to $x = x$ where $\delta = \delta(x)$, the liquid film thickness at any location x is determined to be

$$\delta(x) = \left[\frac{4\mu_l\,k_l(T_{\text{sat}} - T_s)x}{g\rho_l(\rho_l - \rho_v)h_{fg}}\right]^{1/4} \tag{10-18}$$

The heat transfer rate from the vapor to the plate at a location x can be expressed as

$$\dot{q}_x = h_x(T_{\text{sat}} - T_s) = k_l\frac{T_{\text{sat}} - T_s}{\delta} \quad \rightarrow \quad h_x = \frac{k_l}{\delta(x)} \tag{10-19}$$

Substituting the $\delta(x)$ expression from Eq. 10–18, the local heat transfer coefficient h_x is determined to be

$$h_x = \left[\frac{g\rho_l(\rho_l - \rho_v)h_{fg}\,k_l^3}{4\mu_l(T_{\text{sat}} - T_s)x}\right]^{1/4} \tag{10-20}$$

The average heat transfer coefficient over the entire plate is determined from its definition by substituting the h_x relation and performing the integration. It gives

$$h = h_{\text{ave}} = \frac{1}{L}\int_0^L h_x\,dx = \frac{4}{3}h_{x=L} = 0.943\left[\frac{g\rho_l(\rho_l-\rho_v)h_{fg}\,k_l^3}{\mu_l(T_{\text{sat}}-T_s)L}\right]^{1/4} \qquad \textbf{(10-21)}$$

Equation 10–21, which is obtained with the simplifying assumptions stated earlier, provides good insight on the functional dependence of the condensation heat transfer coefficient. However, it is observed to underpredict heat transfer because it does not take into account the effects of the nonlinear temperature profile in the liquid film and the cooling of the liquid below the saturation temperature. Both of these effects can be accounted for by replacing h_{fg} by h_{fg}^* given by Eq. 10–9. With this modification, the *average heat transfer coefficient* for laminar film condensation over a vertical flat plate of height L is determined to be

$$h_{\text{vert}} = 0.943\left[\frac{g\rho_l(\rho_l-\rho_v)h_{fg}^*\,k_l^3}{\mu_l(T_{\text{sat}}-T_s)L}\right]^{1/4} \qquad (\text{W/m}^2\cdot{}^\circ\text{C}),\ 0<\text{Re}<30 \qquad \textbf{(10-22)}$$

where

g = gravitational acceleration, m/s^2
ρ_l, ρ_v = densities of the liquid and vapor, respectively, kg/m^3
μ_l = viscosity of the liquid, kg/m · s
$h_{fg}^* = h_{fg} + 0.68C_{pl}(T_{\text{sat}} - T_s)$ = modified latent heat of vaporization, J/kg
k_l = thermal conductivity of the liquid, W/m · °C
L = height of the vertical plate, m
T_s = surface temperature of the plate, °C
T_{sat} = saturation temperature of the condensing fluid, °C

At a given temperature, $\rho_v \ll \rho_l$ and thus $\rho_l - \rho_v \approx \rho_l$ except near the critical point of the substance. Using this approximation and substituting Eqs. 10–14 and 10–18 at $x = L$ into Eq. 10–8 by noting that $\delta_{x=L} = \dfrac{k_l}{h_{x=L}}$ and $h_{\text{vert}} = \dfrac{4}{3}h_{x=L}$ (Eqs. 10–19 and 10–21) give

$$\text{Re} \cong \frac{4g\rho_l(\rho_l-\rho_v)\delta^3}{3\mu_l^2} = \frac{4g\rho_l^2}{3\mu_l^2}\left(\frac{k_l}{h_{x=L}}\right)^3 = \frac{4g}{3\nu_l^2}\left(\frac{k_l}{3h_{\text{vert}}/4}\right)^3 \qquad \textbf{(10-23)}$$

Then the heat transfer coefficient h_{vert} in terms of Re becomes

$$h_{\text{vert}} \cong 1.47k_l\,\text{Re}^{-1/3}\left(\frac{g}{\nu_l^2}\right)^{1/3}, \qquad \begin{matrix}0<\text{Re}<30\\ \rho_v \ll \rho_l\end{matrix} \qquad \textbf{(10-24)}$$

The results obtained from the theoretical relations above are in excellent agreement with the experimental results. It can be shown easily that using property values in Eqs. 10–22 and 10–24 in the *specified units* gives the condensation heat transfer coefficient in W/m^2 · °C, thus saving one from having

$$h_{\text{vert}} = \left(\frac{\frac{\text{m}}{\text{s}^2}\frac{\text{kg}}{\text{m}^3}\frac{\text{kg}}{\text{m}^3}\frac{\text{J}}{\text{kg}}\left(\frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}\right)^3}{\frac{\text{kg}}{\text{m}\cdot\text{s}}\cdot{}^\circ\text{C}\cdot\text{m}} \right)^{1/4}$$

$$= \left[\frac{\text{m}}{\text{s}}\frac{1}{\text{m}^6}\frac{\text{W}^3}{\text{m}^3\cdot{}^\circ\text{C}^3}\frac{\text{J}}{{}^\circ\text{C}} \right]$$

$$= \left(\frac{\text{W}^4}{\text{m}^8\cdot{}^\circ\text{C}^4} \right)^{1/4}$$

$$= \text{W/m}^2\cdot{}^\circ\text{C}$$

FIGURE 10–25
Equation 10–22 gives the condensation heat transfer coefficient in $\text{W/m}^2\cdot{}^\circ\text{C}$ when the quantities are expressed in the units specified in their descriptions.

to go through tedious unit manipulations each time (Fig. 10–25). This is also true for the equations below. All properties of the *liquid* are to be evaluated at the film temperature $T_f = (T_{\text{sat}} + T_s)/2$. The h_{fg} and ρ_v are to be evaluated at the saturation temperature T_{sat}.

Wavy Laminar Flow on Vertical Plates

At Reynolds numbers greater than about 30, it is observed that waves form at the liquid–vapor interface although the flow in liquid film remains laminar. The flow in this case is said to be *wavy laminar*. The waves at the liquid–vapor interface tend to increase heat transfer. But the waves also complicate the analysis and make it very difficult to obtain analytical solutions. Therefore, we have to rely on experimental studies. The increase in heat transfer due to the wave effect is, on average, about 20 percent, but it can exceed 50 percent. The exact amount of enhancement depends on the Reynolds number. Based on his experimental studies, Kutateladze (1963, Ref. 15) recommended the following relation for the average heat transfer coefficient in wavy laminar condensate flow for $\rho v \ll \rho_l$ and $30 < \text{Re} < 1800$,

$$h_{\text{vert, wavy}} = \frac{\text{Re}\, k_l}{1.08\,\text{Re}^{1.22} - 5.2}\left(\frac{g}{\nu_l^2}\right)^{1/3}, \quad \begin{matrix} 30 < \text{Re} < 1800 \\ \rho_v \ll \rho_l \end{matrix} \tag{10-25}$$

A simpler alternative to the relation above proposed by Kutateladze (1963, Ref. 15) is

$$h_{\text{vert, wavy}} = 0.8\,\text{Re}^{0.11}\, h_{\text{vert (smooth)}} \tag{10-26}$$

which relates the heat transfer coefficient in wavy laminar flow to that in wave-free laminar flow. McAdams (1954, Ref. 2) went even further and suggested accounting for the increase in heat transfer in the wavy region by simply increasing the heat transfer coefficient determined from Eq. 10–22 for the laminar case by 20 percent. Holman (1990) suggested using Eq. 10–22 for the wavy region also, with the understanding that this is a conservative approach that provides a safety margin in thermal design. In this book we will use Eq. 10–25.

A relation for the Reynolds number in the wavy laminar region can be determined by substituting the h relation in Eq. 10–25 into the Re relation in Eq. 10–11 and simplifying. It yields

$$\text{Re}_{\text{vert, wavy}} = \left[4.81 + \frac{3.70\, L k_l (T_{\text{sat}} - T_s)}{\mu_l h_{fg}^*}\left(\frac{g}{\nu_l^2}\right)^{1/3} \right]^{0.820}, \quad \rho_v \ll \rho_l \tag{10-27}$$

Turbulent Flow on Vertical Plates

At a Reynolds number of about 1800, the condensate flow becomes turbulent. Several empirical relations of varying degrees of complexity are proposed for the heat transfer coefficient for turbulent flow. Again assuming $\rho_v \ll \rho_l$ for simplicity, Labuntsov (1957, Ref. 17) proposed the following relation for the turbulent flow of condensate on *vertical plates*:

$$h_{\text{vert, turbulent}} = \frac{\text{Re}\, k_l}{8750 + 58\,\text{Pr}^{-0.5}\,(\text{Re}^{0.75} - 253)}\left(\frac{g}{\nu_l^2}\right)^{1/3}, \quad \begin{matrix} \text{Re} > 1800 \\ \rho_v \ll \rho_l \end{matrix} \tag{10-28}$$

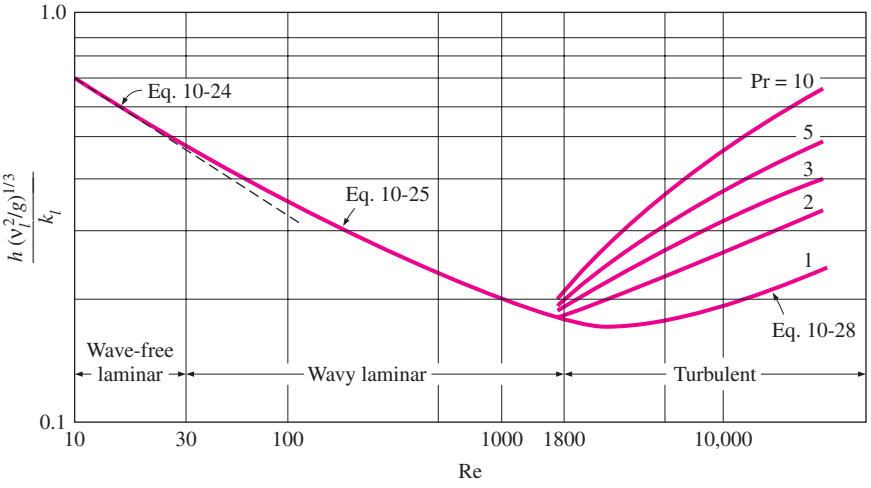


FIGURE 10–26
Nondimensionalized heat transfer coefficients for the wave-free laminar, wavy laminar, and turbulent flow of condensate on vertical plates.

The physical properties of the condensate are again to be evaluated at the film temperature $T_f = (T_{sat} + T_s)/2$. The Re relation in this case is obtained by substituting the h relation above into the Re relation in Eq. 10–11, which gives

$$\mathrm{Re}_{\text{vert, turbulent}} = \left[\frac{0.0690\, Lk_l\, \mathrm{Pr}^{0.5}\,(T_{sat} - T_s)}{\mu_l\, h_{fg}^*}\left(\frac{g}{\nu_l^2}\right)^{1/3} - 151\,\mathrm{Pr}^{0.5} + 253\right]^{4/3} \quad \textbf{(10-29)}$$

Nondimensionalized heat transfer coefficients for the wave-free laminar, wavy laminar, and turbulent flow of condensate on vertical plates are plotted in Figure 10–26.

2 Inclined Plates

Equation 10–12 was developed for vertical plates, but it can also be used for laminar film condensation on the upper surfaces of plates that are *inclined* by an angle θ from the *vertical,* by replacing g in that equation by $g \cos\theta$ (Fig. 10–27). This approximation gives satisfactory results especially for $\theta \leq 60°$. Note that the condensation heat transfer coefficients on vertical and inclined plates are related to each other by

$$h_{\text{inclined}} = h_{\text{vert}}\,(\cos\theta)^{1/4} \qquad \text{(laminar)} \quad \textbf{(10-30)}$$

Equation 10–30 is developed for laminar flow of condensate, but it can also be used for wavy laminar flows as an approximation.

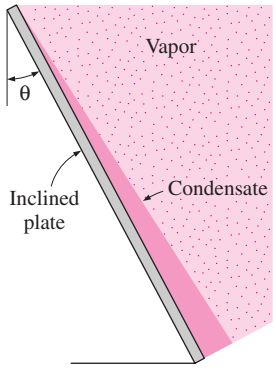


FIGURE 10–27
Film condensation on an inclined plate.

3 Vertical Tubes

Equation 10–22 for vertical plates can also be used to calculate the average heat transfer coefficient for laminar film condensation on the outer surfaces of vertical tubes provided that the tube diameter is large relative to the thickness of the liquid film.

4 Horizontal Tubes and Spheres

Nusselt's analysis of film condensation on vertical plates can also be extended to horizontal tubes and spheres. The average heat transfer coefficient for film condensation on the outer surfaces of a *horizontal tube* is determined to be

$$h_{\text{horiz}} = 0.729 \left[\frac{g\rho_l(\rho_l - \rho_v)\, h_{fg}^{*} k_l^3}{\mu_l (T_{\text{sat}} - T_s) D} \right]^{1/4} \quad (\text{W/m}^2 \cdot {}^\circ\text{C}) \qquad \textbf{(10-31)}$$

where D is the diameter of the horizontal tube. Equation 10–31 can easily be modified for a *sphere* by replacing the constant 0.729 by 0.815.

A comparison of the heat transfer coefficient relations for a vertical tube of height L and a horizontal tube of diameter D yields

$$\frac{h_{\text{vert}}}{h_{\text{horiz}}} = 1.29 \left(\frac{D}{L} \right)^{1/4} \qquad \textbf{(10-32)}$$

Setting $h_{\text{vertical}} = h_{\text{horizontal}}$ gives $L = 1.29^4\, D = 2.77D$, which implies that for a tube whose length is 2.77 times its diameter, the average heat transfer coefficient for laminar film condensation will be the *same* whether the tube is positioned horizontally or vertically. For $L > 2.77D$, the heat transfer coefficient will be higher in the horizontal position. Considering that the length of a tube in any practical application is several times its diameter, it is common practice to place the tubes in a condenser *horizontally* to *maximize* the condensation heat transfer coefficient on the outer surfaces of the tubes.

5 Horizontal Tube Banks

Horizontal tubes stacked on top of each other as shown in Figure 10–28 are commonly used in condenser design. The average thickness of the liquid film at the lower tubes is much larger as a result of condensate falling on top of them from the tubes directly above. Therefore, the average heat transfer coefficient at the lower tubes in such arrangements is smaller. Assuming the condensate from the tubes above to the ones below drain smoothly, the average film condensation heat transfer coefficient for all tubes in a vertical tier can be expressed as

$$h_{\text{horiz, } N \text{ tubes}} = 0.729 \left[\frac{g\rho_l(\rho_l - \rho_v)\, h_{fg}^{*} k_l^3}{\mu_l (T_{\text{sat}} - T_s)\, ND} \right]^{1/4} = \frac{1}{N^{1/4}}\, h_{\text{horiz, 1 tube}} \qquad \textbf{(10-33)}$$

Note that Eq. 10–33 can be obtained from the heat transfer coefficient relation for a horizontal tube by replacing D in that relation by ND. This relation does not account for the increase in heat transfer due to the ripple formation and turbulence caused during drainage, and thus generally yields conservative results.

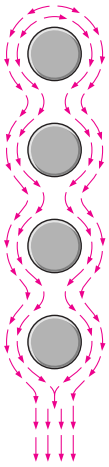


FIGURE 10–28
Film condensation on a vertical tier of horizontal tubes.

Effect of Vapor Velocity

In the analysis above we assumed the vapor velocity to be small and thus the vapor drag exerted on the liquid film to be negligible, which is usually the case. However, when the vapor velocity is high, the vapor will "pull" the liquid at the interface along since the vapor velocity at the interface must drop to the value of the liquid velocity. If the vapor flows downward (i.e., in the same direction as the liquid), this additional force will increase the average velocity of the liquid and thus decrease the film thickness. This, in turn, will decrease the thermal resistance of the liquid film and thus increase heat transfer. Upward vapor flow has the opposite effects: the vapor exerts a force on the

liquid in the opposite direction to flow, thickens the liquid film, and thus decreases heat transfer. Condensation in the presence of high vapor flow is studied [e.g., Shekriladze and Gomelauri (1966), Ref. 23] and heat transfer relations are obtained, but a detailed analysis of this topic is beyond the scope of this introductory text.

The Presence of Noncondensable Gases in Condensers

Most condensers used in steam power plants operate at pressures well below the atmospheric pressure (usually under 0.1 atm) to maximize cycle thermal efficiency, and operation at such low pressures raises the possibility of air (a noncondensable gas) leaking into the condensers. Experimental studies show that the presence of noncondensable gases in the vapor has a detrimental effect on condensation heat transfer. Even small amounts of a noncondensable gas in the vapor cause significant drops in heat transfer coefficient during condensation. For example, the presence of less than 1 percent (by mass) of air in steam can reduce the condensation heat transfer coefficient by more than half. Therefore, it is common practice to periodically vent out the noncondensable gases that accumulate in the condensers to ensure proper operation.

The drastic reduction in the condensation heat transfer coefficient in the presence of a noncondensable gas can be explained as follows: When the vapor mixed with a noncondensable gas condenses, only the noncondensable gas remains in the vicinity of the surface (Fig. 10–29). This gas layer acts as a *barrier* between the vapor and the surface, and makes it difficult for the vapor to reach the surface. The vapor now must diffuse through the noncondensable gas first before reaching the surface, and this reduces the effectiveness of the condensation process.

Experimental studies show that heat transfer in the presence of a noncondensable gas strongly depends on the nature of the vapor flow and the flow velocity. As you would expect, a *high flow velocity* is more likely to remove the stagnant noncondensable gas from the vicinity of the surface, and thus *improve* heat transfer.

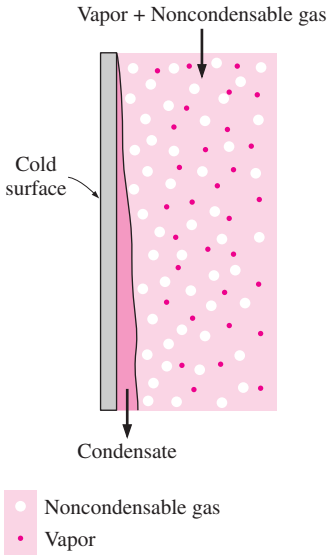


FIGURE 10–29
The presence of a noncondensable gas in a vapor prevents the vapor molecules from reaching the cold surface easily, and thus impedes condensation heat transfer.

EXAMPLE 10–4 Condensation of Steam on a Vertical Plate

Saturated steam at atmospheric pressure condenses on a 2-m-high and 3-m-wide vertical plate that is maintained at 80°C by circulating cooling water through the other side (Fig. 10–30). Determine (*a*) the rate of heat transfer by condensation to the plate and (*b*) the rate at which the condensate drips off the plate at the bottom.

SOLUTION Saturated steam at 1 atm condenses on a vertical plate. The rates of heat transfer and condensation are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The plate is isothermal. **3** The condensate flow is wavy-laminar over the entire plate (will be verified). **4** The density of vapor is much smaller than the density of liquid, $\rho_v \ll \rho_l$.

Properties The properties of water at the saturation temperature of 100°C are $h_{fg} = 2257 \times 10^3$ J/kg and $\rho_v = 0.60$ kg/m^3. The properties of liquid water at the film temperature of $T_f = (T_{sat} + T_s)/2 = (100 + 80)/2 = 90$°C are (Table A-9)

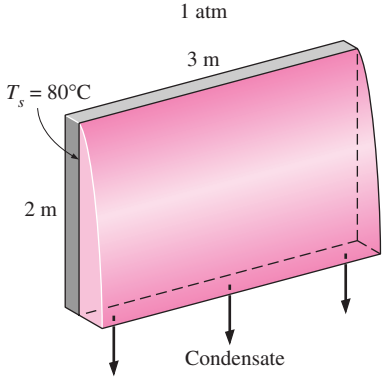


FIGURE 10–30
Schematic for Example 10–4.

$$\rho_l = 965.3 \text{ kg/m}^3 \qquad C_{pl} = 4206 \text{ J/kg} \cdot ^\circ\text{C}$$
$$\mu_l = 0.315 \times 10^{-3} \text{ kg/m} \cdot \text{s} \qquad k_l = 0.675 \text{ W/m} \cdot ^\circ\text{C}$$
$$\nu_l = \mu_l/\rho_l = 0.326 \times 10^{-6} \text{ m}^2/\text{s}$$

Analysis (*a*) The modified latent heat of vaporization is

$$\begin{aligned} h_{fg}^* &= h_{fg} + 0.68C_{pl}(T_{\text{sat}} - T_s) \\ &= 2257 \times 10^3 \text{ J/kg} + 0.68 \times (4206 \text{ J/kg} \cdot ^\circ\text{C})(100 - 80)^\circ\text{C} \\ &= 2314 \times 10^3 \text{ J/kg} \end{aligned}$$

For wavy-laminar flow, the Reynolds number is determined from Eq. 10–27 to be

$$\begin{aligned} \text{Re} = \text{Re}_{\text{vertical, wavy}} &= \left[4.81 + \frac{3.70\,Lk_l(T_{\text{sat}} - T_s)}{\mu_l h_{fg}^*}\left(\frac{g}{\nu_l^2}\right)^{1/3}\right]^{0.820} \\ &= \left[4.81 + \frac{3.70(3 \text{ m})(0.675 \text{ W/m} \cdot ^\circ\text{C})(100 - 90)^\circ\text{C}}{(0.315 \times 10^{-3} \text{ kg/m} \cdot \text{s})(2314 \times 10^3 \text{ J/kg})} \right. \\ &\quad \left. \times \left(\frac{9.81 \text{ m/s}^2}{(0.326 \times 10^{-6} \text{ m}^2/\text{s})^2}\right)^{1/3}\right]^{0.82} \\ &= 1287 \end{aligned}$$

which is between 30 and 1800, and thus our assumption of wavy laminar flow is verified. Then the condensation heat transfer coefficient is determined from Eq. 10–25 to be

$$\begin{aligned} h = h_{\text{vertical, wavy}} &= \frac{\text{Re}\,k_l}{1.08\,\text{Re}^{1.22} - 5.2}\left(\frac{g}{\nu_l^2}\right)^{1/3} \\ &= \frac{1287 \times (0.675 \text{ W/m} \cdot ^\circ\text{C})}{1.08(1287)^{1.22} - 5.2}\left(\frac{9.81 \text{ m/s}^2}{(0.326 \times 10^{-6} \text{ m}^2/\text{s})^2}\right)^{1/3} = 5848 \text{ W/m}^2 \cdot ^\circ\text{C} \end{aligned}$$

The heat transfer surface area of the plate is $A_s = \text{W} \times \text{L} = (3 \text{ m})(2 \text{ m}) = 6 \text{ m}^2$. Then the rate of heat transfer during this condensation process becomes

$$\dot{Q} = hA_s(T_{\text{sat}} - T_s) = (5848 \text{ W/m}^2 \cdot ^\circ\text{C})(6 \text{ m}^2)(100 - 80)^\circ\text{C} = \mathbf{7.02 \times 10^5 \text{ W}}$$

(*b*) The rate of condensation of steam is determined from

$$\dot{m}_{\text{condensation}} = \frac{\dot{Q}}{h_{fg}^*} = \frac{7.02 \times 10^5 \text{ J/s}}{2314 \times 10^3 \text{ J/kg}} = \mathbf{0.303 \text{ kg/s}}$$

That is, steam will condense on the surface at a rate of 303 grams per second.

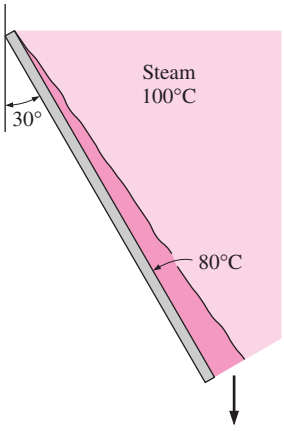


FIGURE 10–31
Schematic for Example 10–5.

EXAMPLE 10–5 **Condensation of Steam on a Tilted Plate**

What would your answer be to the preceding example problem if the plate were tilted 30° from the vertical, as shown in Figure 10–31?

SOLUTION (*a*) The heat transfer coefficient in this case can be determined from the vertical plate relation by replacing g by $g \cos \theta$. But we will use Eq. 10–30 instead since we already know the value for the vertical plate from the preceding example:

$$h = h_{\text{inclined}} = h_{\text{vert}} (\cos \theta)^{1/4} = (5848 \text{ W/m}^2 \cdot ^\circ\text{C})(\cos 30^\circ)^{1/4} = 5641 \text{ W/m}^2 \cdot ^\circ\text{C}$$

The heat transfer surface area of the plate is still 6 m^2. Then the rate of condensation heat transfer in the tilted plate case becomes

$$\dot{Q} = hA_s(T_{\text{sat}} - T_s) = (5641 \text{ W/m}^2 \cdot ^\circ\text{C})(6 \text{ m}^2)(100 - 80)^\circ\text{C} = \mathbf{6.77 \times 10^5 \text{ W}}$$

(*b*) The rate of condensation of steam is again determined from

$$\dot{m}_{\text{condensation}} = \frac{\dot{Q}}{h_{fg}^*} = \frac{6.77 \times 10^5 \text{ J/s}}{2314 \times 10^3 \text{ J/kg}} = \mathbf{0.293 \text{ kg/s}}$$

Discussion Note that the rate of condensation decreased by about 3.6 percent when the plate is tilted.

EXAMPLE 10–6 Condensation of Steam on Horizontal Tubes

The condenser of a steam power plant operates at a pressure of 7.38 kPa. Steam at this pressure condenses on the outer surfaces of horizontal pipes through which cooling water circulates. The outer diameter of the pipes is 3 cm, and the outer surfaces of the pipes are maintained at 30°C (Fig. 10–32). Determine (*a*) the rate of heat transfer to the cooling water circulating in the pipes and (*b*) the rate of condensation of steam per unit length of a horizontal pipe.

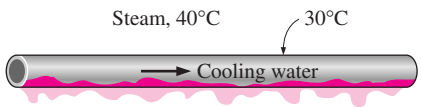


FIGURE 10–32
Schematic for Example 10–6.

SOLUTION Saturated steam at a pressure of 7.38 kPa (Table A-9) condenses on a horizontal tube at 30°C. The rates of heat transfer and condensation are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tube is isothermal.

Properties The properties of water at the saturation temperature of 40°C corresponding to 7.38 kPa are $h_{fg} = 2407 \times 10^3$ J/kg and $\rho_v = 0.05$ kg/m^3. The properties of liquid water at the film temperature of $T_f = (T_{\text{sat}} + T_s)/2 = (40 + 30)/2 = 35^\circ$C are (Table A-9)

$$\rho_l = 994 \text{ kg/m}^3 \qquad C_{pl} = 4178 \text{ J/kg} \cdot ^\circ\text{C}$$
$$\mu_l = 0.720 \times 10^{-3} \text{ kg/m} \cdot \text{s} \qquad k_l = 0.623 \text{ W/m} \cdot ^\circ\text{C}$$

Analysis (*a*) The modified latent heat of vaporization is

$$\begin{aligned} h_{fg}^* &= h_{fg} + 0.68C_{pl}(T_{\text{sat}} - T_s) \\ &= 2407 \times 10^3 \text{ J/kg} + 0.68 \times (4178 \text{ J/kg} \cdot ^\circ\text{C})(40 - 30)^\circ\text{C} \\ &= 2435 \times 10^3 \text{ J/kg} \end{aligned}$$

Noting that $\rho_v \ll \rho_l$ (since $0.05 \ll 994$), the heat transfer coefficient for condensation on a single horizontal tube is determined from Eq. 10–31 to be

$$h = h_{\text{horizontal}} = 0.729\left[\frac{g\rho_l(\rho_l - \rho_v)\, h_{fg}^* k_l^3}{\mu(T_{\text{sat}} - T_s)\, D}\right]^{1/4} \cong 0.729\left[\frac{g\rho_l^2\, h_{fg}^* k_l^3}{\mu_1\,(T_{\text{sat}} - T_s)\, D}\right]^{1/4}$$
$$= 0.729\left[\frac{(9.81\ \text{m/s}^2)(994\ \text{kg/m}^3)^2\,(2435 \times 10^3\ \text{J/kg})(0.623\ \text{W/m} \cdot {}^\circ\text{C})^3}{(0.720 \times 10^{-3}\ \text{kg/m} \cdot \text{s})(40 - 30)^\circ\text{C}(0.03\ \text{m})}\right]^{1/4}$$
$$= 9292\ \text{W/m}^2 \cdot {}^\circ\text{C}$$

The heat transfer surface area of the pipe per unit of its length is $A_s = \pi DL = \pi(0.03\ \text{m})(1\ \text{m}) = 0.09425\ \text{m}^2$. Then the rate of heat transfer during this condensation process becomes

$$\dot{Q} = hA_s(T_{\text{sat}} - T_s) = (9292\ \text{W/m}^2 \cdot {}^\circ\text{C})(0.09425\ \text{m}^2)(40 - 30)^\circ\text{C} = \mathbf{8758\ W}$$

(*b*) The rate of condensation of steam is

$$\dot{m}_{\text{condensation}} = \frac{\dot{Q}}{h_{fg}^*} = \frac{8578\ \text{J/s}}{2435 \times 10^3\ \text{J/kg}} = \mathbf{0.00360\ kg/s}$$

Therefore, steam will condense on the horizontal tube at a rate of 3.6 g/s or 12.9 kg/h per meter of its length.

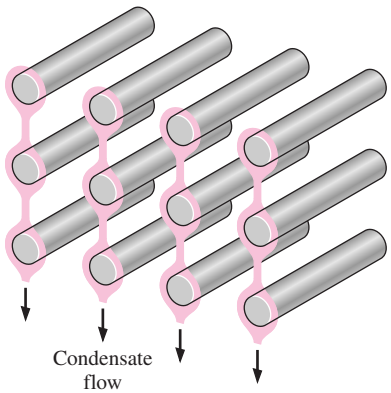


FIGURE 10–33
Schematic for Example 10–7.

EXAMPLE 10–7 **Condensation of Steam on Horizontal Tube Banks**

Repeat the proceeding example problem for the case of 12 horizontal tubes arranged in a rectangular array of 3 tubes high and 4 tubes wide, as shown in Figure 10–33.

SOLUTION (*a*) Condensation heat transfer on a tube is not influenced by the presence of other tubes in its neighborhood unless the condensate from other tubes drips on it. In our case, the horizontal tubes are arranged in four vertical tiers, each tier consisting of 3 tubes. The average heat transfer coefficient for a vertical tier of N horizontal tubes is related to the one for a single horizontal tube by Eq. 10–33 and is determined to be

$$h_{\text{horiz, N tubes}} = \frac{1}{N^{1/4}}\, h_{\text{horiz, 1 tube}} = \frac{1}{3^{1/4}}\,(9292\ \text{W/m}^2 \cdot {}^\circ\text{C}) = 7060\ \text{W/m}^2 \cdot {}^\circ\text{C}$$

Each vertical tier consists of 3 tubes, and thus the heat transfer coefficient determined above is valid for each of the four tiers. In other words, this value can be taken to be the average heat transfer coefficient for all 12 tubes.

The surface area for all 12 tubes per unit length of the tubes is

$$A_s = N_{\text{total}}\, \pi DL = 12\pi(0.03\ \text{m})(1\ \text{m}) = 1.1310\ \text{m}^2$$

Then the rate of heat transfer during this condensation process becomes

$$\dot{Q} = hA_s(T_{\text{sat}} - T_s) = (7060\ \text{W/m}^2 \cdot {}^\circ\text{C})(1.131\ \text{m}^2)(40 - 30)^\circ\text{C} = \mathbf{79{,}850\ W}$$

(*b*) The rate of condensation of steam is again determined from

$$\dot{m}_{\text{condensation}} = \frac{\dot{Q}}{h_{fg}^*} = \frac{79{,}850 \text{ J/s}}{2435 \times 10^3 \text{ J/kg}} = \mathbf{0.0328 \text{ kg/s}}$$

Therefore, steam will condense on the horizontal pipes at a rate of 32.8 g/s per meter length of the tubes.

10–6 ▪ FILM CONDENSATION INSIDE HORIZONTAL TUBES

So far we have discussed film condensation on the *outer surfaces* of tubes and other geometries, which is characterized by negligible vapor velocity and the unrestricted flow of the condensate. Most condensation processes encountered in refrigeration and air-conditioning applications, however, involve condensation on the *inner surfaces* of horizontal or vertical tubes. Heat transfer analysis of condensation inside tubes is complicated by the fact that it is strongly influenced by the vapor velocity and the rate of liquid accumulation on the walls of the tubes (Fig. 10–34).

For *low vapor velocities,* Chato recommends this expression for condensation

$$h_{\text{internal}} = 0.555\left[\frac{g\rho_l(\rho_l - \rho_v)k_l^3}{\mu_l(T_{\text{sat}} - T_s)}\left(h_{fg} + \frac{3}{8}C_{pl}(T_{\text{sat}} - T_s)\right)\right]^{1/4} \quad \textbf{(10-34)}$$

for

$$\text{Re}_{\text{vapor}} = \left(\frac{\rho_v V_v D}{\mu_v}\right)_{\text{inlet}} < 35{,}000 \quad \textbf{(10-35)}$$

where the Reynolds number of the vapor is to be evaluated at the tube *inlet* conditions using the internal tube diameter as the characteristic length. Heat transfer coefficient correlations for higher vapor velocities are given by Rohsenow.

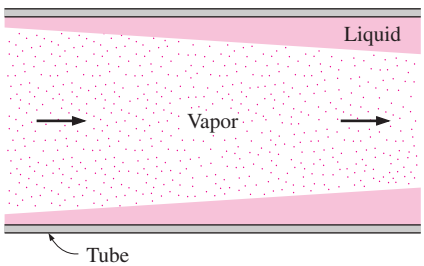


FIGURE 10–34
Condensate flow in a horizontal tube with large vapor velocities.

10–7 ▪ DROPWISE CONDENSATION

Dropwise condensation, characterized by countless droplets of varying diameters on the condensing surface instead of a continuous liquid film, is one of the most effective mechanisms of heat transfer, and extremely large heat transfer coefficients can be achieved with this mechanism (Fig. 10–35).

In dropwise condensation, the small droplets that form at the nucleation sites on the surface grow as a result of continued condensation, coalesce into large droplets, and slide down when they reach a certain size, clearing the surface and exposing it to vapor. There is no liquid film in this case to resist heat transfer. As a result, with dropwise condensation, heat transfer coefficients can be achieved that are more than 10 times larger than those associated with film condensation. Large heat transfer coefficients enable designers to achieve a specified heat transfer rate with a smaller surface area, and thus a smaller (and

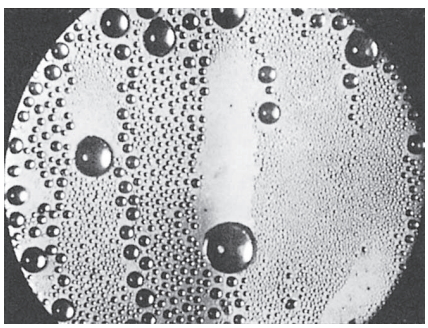


FIGURE 10–35
Dropwise condensation of steam on a vertical surface (from Hampson and Özişik, Ref. 11).